Methodological recommendations for preparation for the Financial University Youth International Mathematical Competition

1. Competition assignment composition and knowledge assessment

Each mathematical competition assignment consists of 8 mathematical problems. The participants should resolve the problems in 90 minutes.

The grading scale of 0 to 100 is used to assess the knowledge. The number of grades depends on the degree of correctness of the math problem solution. A certain number of grades is assigned for each correctly solved problem (see table below).

Problem number	Maximal number of grades that could be assigned
1	9
2	9
3	9
4	13
5	13
6	13
7	17
8	17
Total:	100

You can find the general knowledge assessment principles below.

Problem solving (maximal number of grades assigned: 9)

Grades	Degree of correctness in problem solving
9	A correct solution
7-8	A basically correct solution, with a few minor mistakes that do not affect the result.
5-6	The solution contains minor mistakes; there are errors in the rationale behind the solution. It is basically a correct solution but it requires a few amendments/added components.
4	A correct solution of one of the two significant cases described.
2-3	The auxiliary assertions that help in solving the problem are proved.
1	Some important cases are considered, but there is no proposed solution (or the solution proposed is erroneous).
0	The solution is incorrect, there is no progress in solving the problem, or there is no solution.

Problem solving (maximal number of grades assigned: 13)

Grades	Degree of correctness in problem solving
13	A correct solution
10-12	A basically correct solution, with a few minor mistakes that do not affect the result.
7-9	The solution contains minor mistakes; there are errors in the rationale behind the solution. It is basically a correct solution but it requires a few amendments/added components.

6	A correct solution of one of the two significant cases described.
3-5	The auxiliary assertions that help in solving the problem are proved.
1-2	Some important cases are considered, but there is no proposed solution (or the solution proposed is erroneous).
0	The solution is incorrect, there is no progress in solving the problem, or there is no solution.

Problem solving (maximal number of grades assigned: 17)

Grades	Degree of correctness in problem solving
17	A correct solution
13-16	A basically correct solution, with a few minor mistakes that do not affect the result.
9-12	The solution contains minor mistakes; there are errors in the rationale behind the solution. It is basically a correct solution but it requires a few amendments/added components.
8	A correct solution of one of the two significant cases described.
4-7	The auxiliary assertions that help in solving the problem are proved.
1-3	Some important cases are considered, but there is no proposed solution (or the solution proposed is erroneous).
0	The solution is incorrect, there is no progress in solving the problem, or there is no solution.

2. Topics for examining before the competition

Numbers and calculations. Interests and shares.
Expressions and their transformations.
Equations and inequations.
Diophantine equations
The Dirichlet's principle
Set theory. Combinatorics.
Mathematical logic. Method of mathematical induction.
Mathematical games.
Graphs.
Sequences.
Trigonometry.
Functions.
Planimetry. Stereometry. Coordinate methods.
Probability theory.

3. Problems for drilling

1. Calculate the largest root of the following equation: $|x-2015|=|x-2017|$.

2. Calculate the sum of all integer solutions to the following inequation: $\frac{f'(x)}{g'(x)} \le \frac{3}{2}x+4$,

where $f(x)=x^3+3x^2-7$, $g(x)=x^2-4x+8$.

3. A circle is inscribed into a square of 60 square centimeters into which the second square is inscribed into which the second circle is inscribed into which the third square is inscribed. Find the area of the third square.

4. There are 30 cities in the country. They are connected by roads. There are 2 roads that go out of each of the 8 cities, and there are 7 roads that go out of each of the rest of the cities. How many roads are there in the country?

5. A party was attended by 11 couples. Each of the men shook hands with every female present except for his wife. Women greeted each other without shaking hands. How many handshakes were there in total?

6. How many natural numbers-based solutions does the following equation have? $x^4y^4 - 10x^2y^2 + 9 = 0$.

7. The average grade for the examination in two subgroups is 4.0, the average score in the first subgroup is 3.6, and the average score in the second subgroup is 4.2. What could be the smallest number of students in each subgroup?

8. Calculate the meaning of the expression: $\sqrt[3]{2017^2 + 2017 \cdot 2018 + 2018^2 + 2017^3}$.

9. On the coordinate plane, draw a figure consisting of points whose coordinates satisfy:

$$\left||x| - 2\right| + |y - 3| \le 3.$$

Calculate the area of the figure.

10. Two candles of the same length were lit at night at the same time. The first candle burns completely in 3 hours, and the second one, in 4 hours. At four o'clock in the morning, it was discovered that the length of one of the candles was three times as much as the length of the second candle. Calculate the time when the candles were lit, given that they burn with the same speed.

11. The points K, L and M, respectively, are taken on the sides AB, BC and CA of the ABC triangle. It is known that $AK : KB = BL : LC = CM : AM = 1 : 7$. Calculate the area of the KLM triangle if the area of the ABC triangle is equal to 64.

12. Calculate the sum of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{2025}+\sqrt{2024}}$.

13. Figures 1, 2, 3, 4 consist of identical squares and have areas of 1, 5, 13 and 25, respectively. Continue the sequence of figures and calculate the area of the 100th figure.

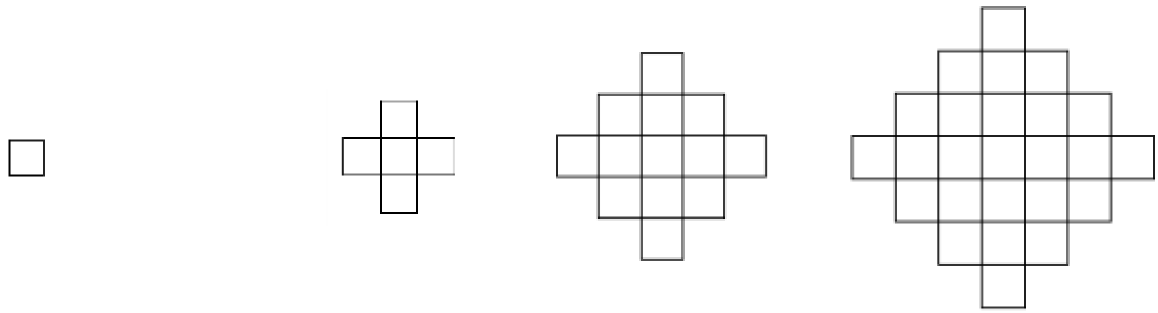

Figure 1 Figure 2 Figure 3 Figure 4

14. Calculate the remainder of dividing of $2016^{2017} - 2017^{2016}$ by 11.

15. Prove that during any football match there was a moment when one of the teams scored as many goals as the other one had yet to score.

16. Calculate the positive values of x, y and z when the smallest value of $x^2 - y^2z + \frac{1}{16} + \frac{y^8}{4x^2} + z^4$ is reached. When answering the question, specify the value of the expression $64x^4 + 6y^2 - 4z$.

4. Useful links for the drills

www.problems.ru
www.mathkang.ru
zadachi.mccme.ru
artofproblemsolving.com